Section A:

CLASSICAL RELATIVITY

"*Two things awe me most — the starry sky above me,*

and the moral law within me"

Immanuel Kant (1724–1804)

CHAPTER 1

SUCCESS OF RELATIVITY

1.1 Einstein's Idea

In 1905, Einstein published his so-called special theory of relativity. He introduced two postulates to physics. First was the principle of special relativity; that is, all inertial frames are equivalent for the description of all physical laws. Second was the principle of the constancy of the speed of light; that is, the speed of the light is the same for all observers. These two postulates changed the Galilean transformation to the Lorentz transformation. That is

$$x' = x - vt \Longrightarrow x' = k\,(x - vt) \qquad 1\text{-}1\text{-}1$$

$$y' = y \Longrightarrow y' = y \qquad 1\text{-}1\text{-}2$$

$$z' = z \Longrightarrow z' = z \qquad 1\text{-}1\text{-}3$$

$$t' = t \Longrightarrow t' = k\frac{t - vx}{c^2} \qquad 1\text{-}1\text{-}4$$

It also changed traditional quantities to relativistic quantities. That is,

$m = km_0$ (mass increase) 1-1-5

$t = kt_0$ (time delay) 1-1-6

$l = \frac{l_0}{k}$ (length contraction) 1-1-7

$E = mc^2$ (relativistic energy) 1-1-8

$$k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ (} k\text{- factor)} \qquad 1\text{-}1\text{-}9$$

Because the validation and accuracy of this formula were proven by a large number of experiments and observations, it has become accepted almost as fact and without doubt. As discussed by Clifford M. Will in his book "Was Einstein Right?," it was after the British theorist P.A.M. Dirac had merged special relativity with quantum theory in 1928 that the confirmation of theoretical predictions with experiments could become a reality. Revelations of such phenomena as electron spins and of anti matter, among others, resulted. Dirac's theory follows: The wave equation of the electron is

$$\{P_0 - (m^2c^2 + P_1{}^2 + P_2{}^2 + P_3{}^2)^{1/2})\}\psi = 0 \qquad 1\text{-}1\text{-}10$$

$$\{P_0{}^2 - m^2c^2 - P_1{}^2 - P_2{}^2 - P_3{}^2\}\psi = 0 \qquad 1\text{-}1\text{-}11$$

$$\{P_0 - \alpha_1 P_1 - \alpha_2 P_2 - \alpha_3 P_3 - \beta\}\psi = 0 \qquad 1\text{-}1\text{-}12$$

$$\{P_0 - \rho_1(\sigma, P) - \rho_3 mc\}\psi = 0 \qquad 1\text{-}1\text{-}13$$

$$\{P_0 + \frac{e}{c}A_0 - \rho_1 \ (\sigma,\ P + \frac{e}{c}) - \rho_3 mc\} \ \psi = 0 \qquad 1\text{-}1\text{-}14$$

$$\overline{\psi}^{\dagger}\{P_0 + \frac{e}{c}A_0 - \rho_1 \ (\sigma,\ P + \frac{e}{c}A) - \rho_3 mc\} = 0 \qquad 1\text{-}1\text{-}15$$

Existence of spin is

$$\{(P_0 + \frac{e}{c}A_0)^2 - (P + \frac{e}{c}A)^2 - m^2c^2\} \ \psi = 0 \qquad 1\text{-}1\text{-}16$$

$$\{(P_0+\frac{e}{c}A_0)^2-(\sigma,P+\frac{e}{c}A)^2-m^2c^2-\rho_1[(P_0+\frac{e}{c}A_0)(\sigma,P+\frac{e}{c}A)$$

$$-(\sigma,P+\frac{e}{c}A)(P_0+\frac{e}{c}A_0)]\}\ \psi=0 \qquad 1\text{-}1\text{-}17$$

$$(P+\frac{e}{c}A)\times(P+\frac{e}{c}A)=\frac{e}{c}\{P\times A+A\times P\}=-\frac{i\hbar e}{c}\cdot\mathrm{curl}A=-\frac{i\hbar e}{c}\cdot\mathcal{H}$$

……………………………………………………………………1-1-18

$$(\sigma,P+\frac{e}{c}A)^2=(P+\frac{e}{c}A)^2+\frac{\hbar e}{c}(\sigma,\mathcal{H}) \qquad 1\text{-}1\text{-}19$$

$$(P_0+\frac{e}{c}A_0)(\sigma,P+\frac{e}{c}A)-(\sigma,P+\frac{e}{c}A)(P_0+A_0)$$

$$=\frac{i\hbar e}{c}(\sigma,\frac{1dA}{cdt}+\mathrm{grad}A_0)$$

$$=-i\frac{\hbar e}{c}(\sigma,\varepsilon) \qquad 1\text{-}1\text{-}20$$

$$\{(P_0+\frac{e}{c}A_0)^2-(P+\frac{e}{c}A)^2-mc^2$$

$$-\frac{\hbar e}{c}(\sigma,\mathcal{H})+i\rho_1\frac{\hbar e}{c}(\sigma,\varepsilon)\}\ \psi=0 \qquad 1\text{-}1\text{-}21$$

$$H=-eA_0+c\rho_1\left(\sigma,P+\frac{e}{c}A\right)+\rho_3mc^2 \qquad 1\text{-}1\text{-}22$$

$$(\frac{H}{c}+\frac{e}{c}A_0)^2=\{\rho_1\left(\sigma,P+\frac{e}{c}A\right)+\rho_3mc\}^2 \qquad 1\text{-}1\text{-}23$$

$$H_1+eA_0=\frac{1}{2m}(P+\frac{e}{c}A)^2+\frac{\hbar e}{2mc}(\sigma,\mathcal{H}) \qquad 1\text{-}1\text{-}24$$

$$i\hbar\dot{\sigma}_1=\sigma_1H-H\sigma_1$$

$$=c\rho_1\{\sigma_1(\sigma,P)-(\sigma,P)\sigma_1\}$$

$$=c\rho_1(\sigma_1\sigma-\sigma\sigma_1,P)$$

$$= 2ic\,\rho_1\{\sigma_3 P_2 - \sigma_2 P_3\} \qquad 1\text{-}1\text{-}25$$

$$\dot{m}_1 + \frac{1}{2}\hbar\dot{\sigma}_1 = 0 \qquad 1\text{-}1\text{-}26$$

So the vector $(\dot{m} + \frac{1}{2}\hbar\sigma)$ is a constant of the motion.

This result can be interpreted by saying the electron has a spin angular momentum

$$\mu_s = -g_s\mu_b S/\hbar, \qquad S = \sqrt{s(s+\tfrac{1}{2})}\,\hbar, \qquad g_s = 2$$

The real observational value by Lamb is $g = 2.002319304$.

This was a great triumph for relativity theory, as was expressed by Clifford M. Will in the following excerpt from his book.

> "The observational evidence for time dilation is overwhelming. Time dilation slows down the decay rates of the unstable muon generated by cosmic rays allowing them to reach sea level. Quantitative tests of time dilation have been performed numerous times at particle accelerators. A classical experiment was performed in 1966 at the accelerator in CERN. Muons produced by collision at one of the targets in the accelerator were deflected by a magnet, so that they would move in circular paths. When started for a decent interval, their speed reached 99.7% of the velocity of light and the observed 12-fold increase in their lifetime agreed with the predicted value with 2% accuracy." [1]

The first experimental confirmation of the predictions of relativity theory concerning the defense of mass on velocity was provided by Bücherer in 1909. He applied to electrons of high velocity a variation of the technique used by Thomson to measure the charge-to-mass ratio of slowly moving electrons. Bücherer's results are shown by the crosses in Figure 1-1-1, where some more extensive results are shown by dots and predictions of $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ are shown by the solid curve.

1. Excerpt from Will, Clifford M. 1993. *Was Einstein Right?* 2nd Ed. New York: Basic Books.

These results prove not only that $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ has the correct functional form, but also that the limiting velocity for transmission of information in the theory of relativity is in fact equal to the velocity of light, $c = 2.99792458 \times 10^8$ m/s.

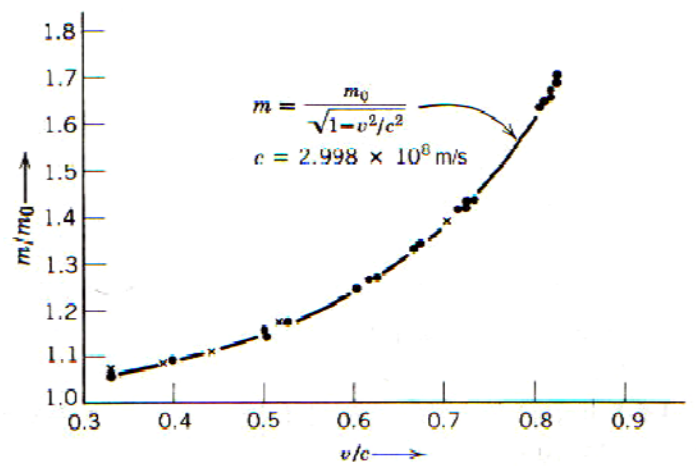

Figure 1-1-1

This theory is so much a part not only of physics but also of everyday life that it is no longer appropriate to view it as the special theory of relativity. It is in fact as basic to the world as the existence of the atom or the quantum theory of matter. It has been tested directly time and time again and has provided the foundation for modern advances in elementary particle physics. Because of such accurate experimental results, the special relativity theory is accepted almost as fact in modern physics. Meanwhile, in 1915, Einstein published his general theory of relativity. He introduced two postulates to physics. First was the principle of general relativity: all observers are equivalent.

The second postulate was the principle of equivalence: gravitational mass equals inertial mass.

Under this theory, gravitational mass bends its surrounding space-time. Its field equation is

$$G_{\mu\nu} = \frac{8\pi G}{C^4} T_{\mu\nu}, \quad T_{\mu\nu} = \rho u^{\mu}\mu^{\nu}, \quad \rho = \gamma^2 \rho_0, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

With the cosmological constant, the field equation is

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Prediction of this theory can be tested by several methods.

One method involves the deflection of light by the sun.

The other method involves the precession of perihelia.

(1) Unbound orbits: [2] deflection of light by the sun

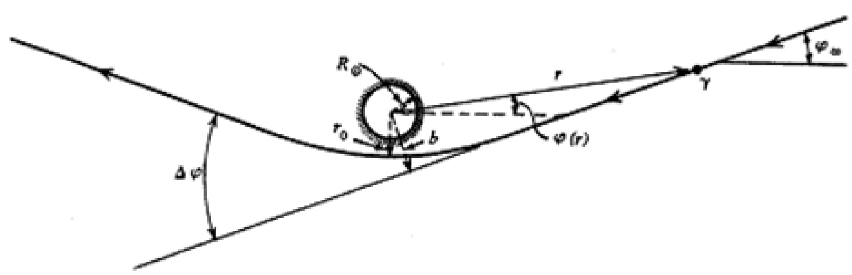

Figure 1-1-2 (Source: Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, p. 189. Reproduced with permission from John Wiley & Sons, Inc. © 1972.)

Consider a particle or photon approaching the sun from very great distances. At infinity the metric becomes Minkowskian. That is, $A(\infty) = B(\infty) = 1$ and motion is expected to occur on a straight line at constant velocity v.

That is,

$$b \cong r \sin(\varphi - \varphi_\infty) \cong r(\varphi - \varphi_\infty) \qquad 1\text{-}1\text{-}27$$

2. Equations 1-1-27 through 1-1-40 in the following pages extracted from Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, pp. 189–191. Reproduced with permission from John Wiley & Sons, Inc. © 1972.

$$-v \cong \frac{d}{dt}(r \cos(\varphi - \varphi_\infty)) \cong \frac{dr}{dt} \qquad 1\text{-}1\text{-}28$$

The constants of the motion are

$$J = bv^2, \quad E = 1 - v^2 \qquad 1\text{-}1\text{-}29$$

$$\frac{A(r)}{r^4}\left(\frac{dr}{d\varphi}\right)^2 + \frac{1}{r^2} - \frac{1}{J^2}\frac{1}{B(r)} = -\frac{E}{J^2} \qquad 1\text{-}1\text{-}30$$

$E = 1 - v^2$ gives

$$J = r_0\left(\frac{1}{B(r_0)} - 1 + v^2\right)^{½} \qquad 1\text{-}1\text{-}31$$

The orbit is then described by

$$\varphi(r) = \varphi_\infty + \int_r^\infty \frac{1}{r^2} \frac{A^{½}}{(\frac{1}{r^2}} \frac{(r)}{[\frac{1}{B(r)} - 1 + v^2} \frac{dr}{][\frac{1}{B(r_o)} - 1 +} \frac{1}{v^2]^{-1} - \frac{1}{r^2})^{½}} \qquad 1\text{-}1\text{-}32$$

Hence the deflection of the orbit from a straight line is

$$\Delta\varphi = 2|\varphi_{(r_o)} - \varphi_{(\infty)}| - \pi \qquad 1\text{-}1\text{-}33$$

If $\Delta\varphi$ is negative, then the trajectory is bent away from the sun. For the photon of $v^2 = 1$

$$\varphi_{(r)} - \varphi_\infty = \int_r^\infty A^{½}(r)\left[\left(\frac{r}{r_0}\right)^2\left(\frac{B(r_0)}{B(r)}\right) - 1\right]^{-½}\frac{dr}{r} \qquad 1\text{-}1\text{-}34$$

$$A(r) = 1 + 2\gamma\left(\frac{MG}{r}\right) + \ldots \qquad 1\text{-}1\text{-}35$$

$$B(r) = 1 - 2\gamma\left(\frac{MG}{r}\right) + \ldots \qquad 1\text{-}1\text{-}36$$

$$\left(\frac{r}{r_0}\right)^2\left(\frac{B(r_0)}{B(r)}\right) - 1 = \left[\left(\frac{r}{r_0}\right)^2 - 1\right]\left[1 - \frac{2MGr}{r_0(r + r_0)} + \ldots\right] \qquad 1\text{-}1\text{-}37$$

$$\varphi_{(r)} - \varphi_{(\infty)} = \sin^{-1}\left(\frac{r_0}{r}\right) + \frac{MG}{r_0}\left(1 + \gamma - \gamma\sqrt{1 - \left(\frac{r_0}{r}\right)^2} - \sqrt{\frac{r - r_0}{r + r_0}} + \ldots\right) \qquad 1\text{-}1\text{-}38$$

The deflection is

$$\Delta\varphi = \left(\frac{4MG}{r_0}\right)\left(\frac{1+\gamma}{2}\right) \qquad \text{1-1-39}$$

$M = M_{\odot} = 1.97 \times 10^{33}$ g

$MG = M_{\odot}G = 1.475$ km

$r = R_{\odot} = 6.95 \times 10^{5}$ km

$$\Delta\varphi = \left(\frac{R_{\odot}}{r_0}\right)\theta_{\odot} \qquad \text{1-1-40}$$

$$\theta_{\odot} = \frac{4GM_{\odot}}{R_{\odot}}\left(\frac{1+\gamma}{2}\right) = 1.75''\left(\frac{1+\gamma}{2}\right) = 1.75'' \qquad \text{1-1-41}$$

To test this prediction, in the decade from 1964 to 1974, radio telescopes throughout the world were trained towards distant quasars such as 3C273 and 3C279 to determine whether quasar radio waves would bend in the sun's gravitational field. The radio antennas included those at the Goldstone Complex, the Haystack Observatory, the Owen Valley Observatory, the National Radio Astronomy Observatory (NRAO) facilities**,** as well as the Mullard Radio Astronomy Observatory in the United Kingdom, and the Westerbork Synthesis Radio Telescope (WSRT) in The Netherlands, among others. The earlier data obtained at Owen Valley and Goldstone gave 10% and 15% correlation errors with the predicted value (see Table 1-1-2). By 1975, with improved experimental methods for data collection, the correlation error was reduced to only 1%. [3]

Experimental values are provided in Tables 1-1-1 and 1-1-2.

The excellent agreement of the theoretical values with the experimental values is a triumph of general relativity.

3. For a rich account of the history and details of the experiments mentioned in this chapter, readers can refer to "*Was Einstein Right?"* by Clifford M. Will (2^{nd} Ed., New York: Basic Books 1993).

Table 1-1-1. Measurements of the Deflection of Light by the Sun

Eclipse	Site	Number of Stars	$r_0/R_\odot$	$\theta_\odot$(sec)	Ref.
May 29, 1919	Sobral	7	2–6	1.98 ± 0.16	a
	Principe	5	2–6	1.61 ± 0.40	a
September 21, 1922	Australia	11–14	2–10	1.77 ± 0.40	b
	Australia	18	2–10	1.42 to 2.16	c
	Australia	62–85	2.1–14.5	1.72 ± 0.15	d
	Australia	145	2.1–42	1.82 ± 0.20	e
May 9, 1929	Sumatra	17–18	1.5–7.5	2.24 ± 0.10	f
June 19, 1936	U.S.S.R.	16–29	2–7.2	2.73 ± 0.31	g
	Japan	8	4–7	1.28 to 2.13	h
May 20, 1947	Brazil	51	3.3–10.2	2.01 ± 0.27	i
February 25, 1952	Sudan	9–11	2.1–8.6	1.70 ± 0.10	j

Source: Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, p.193. Reproduced with permission from John Wiley & Sons, Inc. © 1972.

Table 1-1-2. Interferometric Measurements of the Deflection of Radio Waves from the Source 3C279 by the Sun

Facility	Radar Frequency (MHz)	Baseline (km)	Period	$\theta_\odot$ (sec)	Ref.
Owens Valley	9602	1.0662	9/30–10/15, 1969	1.77 ± 0.20	a
Goldstone	2388	21.566	10/2–10/10, 1969	$1.82\ ^{+0.24}_{-0.17}$	b
Goldstone/ Haystack	7840	3899.92	9/30–10/15, 1969	1.80 ± 0.2	c
NRAO	2695 & 8085	2.7	10/2–10/12, 1970	1.57 ± 0.08	d
	2697 & 4993.8	1.41	10/8, 1970	1.87 ± 0.3	e

Source: Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, p. 194. Reproduced with permission from John Wiley & Sons, Inc. © 1972.

(2) Bound orbit: [4] precession of perihelia

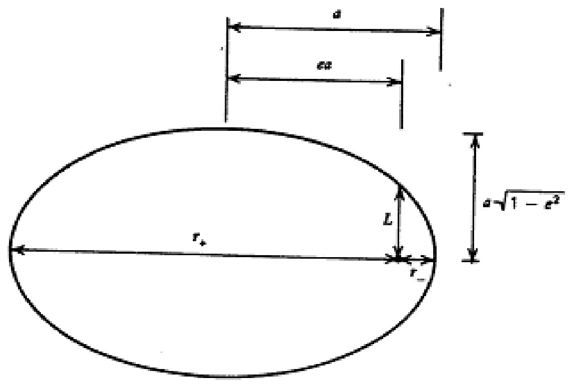

Figure 1-1-3 **(Source: Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, p. 195. Reproduced with permission from John Wiley & Sons, Inc. © 1972.)**

Now consider a test particle bound in an orbit around the sun at perihelion and aphelion. When r reaches its minimum and maximum values, $r-$ and $r+$, respectively, both points $\frac{dr}{d\rho}$ vanish, so

$$\frac{1}{r_{\pm}{}^{2}} - \frac{1}{J^2 B(r_{\pm})} = -\frac{E}{J^2} \qquad \text{1-1-42}$$

$$E = \frac{\frac{r_+^2}{B(r_+)} - \frac{r_-^2}{B(r_-)}}{r_+^2 - r_-^2} \qquad \text{1-1-43}$$

$$J^2 = \frac{\frac{1}{B(r_+)} - \frac{1}{B(r_-)}}{\frac{1}{r_+^2} - \frac{1}{r_-^1}} \qquad \text{1-1-44}$$

$$\varphi(r) = \varphi(r_-) + \int_{r_-}^{r} A^{1/2}(r)\left[\frac{1}{J^2 B(r)} - \frac{E}{J^2} - \frac{1}{r^2}\right]^{-1/2}\left(\frac{dr}{r^2}\right) \qquad \text{1-1-45}$$

$$\Delta\varphi = 2q|\varphi(r_+) - \varphi(r_-)| - 2\pi \qquad \text{1-1-47}$$

4. Equations 1-1-42 through 1-1-61 in the following pages extracted from Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, pp. 194–198. Reproduced with permission from John Wiley & Sons, Inc. © 1972

$$A(r) = 1 + 2\gamma(\frac{MG}{r}) + \ldots \qquad \text{1-1-48}$$

$$B(r) = 1 - \frac{2GM}{r} + \frac{2(\beta-\gamma)M^2G^2}{r^2} + \ldots \qquad \text{1-1-49}$$

$$B^{-1}(r) \backsimeq 1 + \frac{2GM}{r} + \frac{2(2-\beta+\gamma)G^2M^2}{r^2} \qquad \text{1-1-50}$$

$$\frac{r_-^2\left(B^{-1}(r)-B^{-1}(r_-)\right)-r_+^2\left(B^{-1}(r)-B^{-1}(r_+)\right)}{r_+^2 r_-^2\left(B^{-1}(r_+)-B^{-1}(r_-)\right)} - \frac{1}{r^2} = C(\frac{1}{r_-}-\frac{1}{r})(\frac{1}{r}-\frac{1}{r_+}) \qquad \text{1-1-51}$$

$$C = \frac{r_+^2\left(1-B^{-1}(r_+)\right)-r_-^2\left(1-B^{-1}(r_-)\right)}{r_+ r_-\left(B^{-1}(r_+)-B^{-1}(r_-)\right)} \qquad \text{1-1-52}$$

$$C \backsimeq 1 - (2-\beta+\gamma)MG\,(\frac{1}{r_+}-\frac{1}{r_-}) \qquad \text{1-1-53}$$

$$\varphi(r) - \varphi(r_-) = \left[1 + \frac{1}{2}(2-\beta+2\gamma)MG\left(\frac{1}{r_+}-\frac{1}{r_-}\right)\right]\left[\psi + \frac{\pi}{2}\right] - \frac{1}{2}\gamma GM(\frac{1}{r_+}-\frac{1}{r_-})\cos\psi \qquad \text{1-1-54}$$

$$\Delta\varphi = (\frac{6\pi MG}{L})(\frac{2-\beta+2\gamma}{3}) \quad \text{Radian /revolution} \qquad \text{1-1-55}$$

$$\frac{1}{L} \equiv \frac{1}{2}(\frac{1}{r_+}+\frac{1}{r_-}) \qquad \text{1-1-56}$$

$$r_\pm = (1 \pm e)a \qquad \text{1-1-57}$$

$$L = (1 - e^2)a \qquad \text{1-1-58}$$

$$\Delta\varphi = 6\pi\frac{MG}{L} \quad \text{Radian/revolution} \qquad \text{1-1-59}$$

For the planet Mercury, $L = 55.3 \times 10^6$ km, MG = 1,475 km, and $\Delta\varphi = 0.1038''$ per revolution.

Since Mercury makes 415 revolutions per century, the prediction of the general theory of relativity is that

$$\Delta\varphi = 43.03'' \text{ per century} \qquad 1\text{-}1\text{-}60$$

$$\left(\frac{2-\beta+\gamma}{3}\right) = 1.00 \pm 0.01 \qquad 1\text{-}1\text{-}61$$

Experiments to verify this postulate of Einstein's ran between 1964 and 1970, with the first being conducted by Irwin Shapiro via use of the Haystack Observatory when he was still at MIT. By observing the time it took for radar pulses to travel from Venus, and later from Mercury, through the sun's gravitational field toward Earth, the time delay of light could be tested. By 1967, and after more than 400 radar observations, the experimental data from Mercury correlated with the predicted value to within a 20% error.
The experimental values are presented in Table1-1-3.

Table 1-1-3. Comparison of Theoretical and Observed Centennial Precessions of Planetary Orbits

Planet	a (10^6 km)	e	$\frac{6\pi MG}{L}$	Revolutions / Century	$\Delta\varphi$ (seconds/century) Gen. Rel.	Observed
Mercury (☿)	57.91	0.2056	0.1038″	415	43.03	43.11 ± 0.45
Venus (♀)	108.21	0.0068	0.058″	149	8.6	8.4 ± 4.8
Earth (⊕)	149.60	0.0167	0.038″	100	3.8	5.0 ± 1.2
Icarus	161.0	0.827	0.115″	89	10.3	9.8 ± 0.8

Source: Weinberg, Steven. 1972. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, p. 198. Reproduced with permission from John Wiley & Sons, Inc. © 1972.

In 1969 and 1970, the Mariner 6 and 7 spacecrafts were employed to further test Einstein's postulate. Mariner 6 gave a Shapiro time delay of 200 μs, whereas that by Mariner 7 was 180 μs. The data agreed with the predicted value to within 3% error.

Once again, because the theoretical values agree well with the experimental values, it represents yet another triumph of the general theory of relativity.

Robert V. Pound of Harvard University, together with his student Glen A. Rebka, was the first to truly verify Einstein's general relativity theory, in the famous Pound−Rebka experiment conducted in 1960. Applying the Mössbauer effect (i.e., recoil-free emittance and absorbance) into their experimental design, and making use of the ~22.5 m height of the Jefferson Tower of Harvard University to provide the gravitational red shift, Pound and Rebka dropped a sample of unstable ^{57}Fe isotope from the top of the tower to another ^{57}Fe sample at the bottom of the tower and calculated the shifts in frequency of the emitted gamma rays. Special relativity predicts a Doppler red shift of

$$f_r = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} f_e \qquad \text{1-1-62}$$

On the other hand, general relativity predicts a gravitational blue shift of

$$f_r = \sqrt{\frac{1-\frac{2GM}{(R+h)c^2}}{1-\frac{2GM}{RC^2}}} f_e \qquad \text{1-1-63}$$

The detector at the bottom sees a superposition of the two effects. The emitter dish was placed in an elevator and the speed was varied until the two effects cancelled each other, a phenomenon detected by reaching resonance. Mathematically, this is

$$\sqrt{\left(\frac{1-\frac{V}{C}}{1+\frac{v}{c}}\right)\left(\frac{1-\frac{2GM}{(R+h)C^2}}{1-\frac{2GM}{RC^2}}\right)} = 1 \qquad \text{1-1-64}$$

In the Pound–Rebka experiment, $h < R$; therefore,

$$V \approx \frac{gh}{c} = 7.5 \times 10^{-7} \ \text{m/s} \qquad \text{1-1-65}$$

This experimental value obtained by Pound and Rebka agreed with the predicted value to within a 10% error. A further experiment conducted by Pound and J.L. Snider in 1964, using radioactive cobalt as the gamma ray emitting source, improved the prediction accuracy to 1% error. After the historic lunar landing in 1969, the ability to conduct range tests was realized. Retro reflectors placed on the moon could reflect laser pulses fired from Earth, giving precise measurements of the time taken for each pulse to make one round trip between Earth and the moon. In the period between 1971 and 1975, over 1,500 pulse runs were conducted, giving measurements of the distance between Earth and the moon to within 15 cm precision.

These lunar range tests helped to answer a question posed by Kenneth L. Nordtvedt (Montana State University) of whether it was possible for massive celestial bodies to fall with the same acceleration within a specific gravitational field. This so-called “Nordtvedt effect” was verified when the moon and Earth were calculated to fall toward the sun with the same acceleration, within a precision of 7 parts in 10 trillion. This same effect was later found to be true for other celestial bodies with strongly bound internal gravitational energy, such as black bodies and neutron stars. As Clifford M. Will expressed, this was equivalent to saying that “according to general relativity, a black hole and a ball of aluminum fall with the same acceleration.”

The longest research by far to test the postulates of general relativity is the Stanford Relativity Gyroscope Experiment. Conceived in 1956 by Stanford University affiliated professors Leonard I. Schiff, Robert H. Cannon, and William Fairbanks, the challenge was to build a drag-free orbiting gyroscope laboratory with the aim of measuring both the geodetic effect and the frame-dragging effect to an accuracy of half a million arc seconds per year. In March 1964, NASA stepped in to fund the project, naming Cannon and Fairbanks as equal co-project investigators, and Schiffe as the project advisor. Regrettably, Leonard Schiff died in January 1971.

The major events for the Stanford Relativity Gyroscope Experiment, now better known as Gravity Probe B (GP-B), were as follows:

- 20 April 2004, GP-B launched from the van den Berg Air Force Base and successfully inserted into polar orbit

- 27 April 2004, GP-B entered its science phase
- 15 August 2005, calibration phase ended
- February 2006, analysis team realized that more error analysis was required
- December 2006, competition of phase III of data analysis
- 14 April 2007, announcement of best results obtained for data

On that historical day in April, Francis Everitt announced the initial results in a plenary talk at the meeting of the American Physical Society.

The data from the GP-B gyroscopes clearly confirmed Einstein's predicted geodetic effect to a precision of better than 1%. To date, the frame-dragging effect is measured as being 170 times smaller than the geodetic effect, and Stanford scientists are still extracting the signatures from the spacecraft data.

1.2 Appearance of Inconsistency

Despite such brilliant successes, when this theory is applied to the universe, serious contradictions appear between theory and observation:

The Friedman–Lemaitre-Robertson–Walker metric is

$$dS^2 = c^2 dt^2 - a^2(t) d\Sigma^2 \qquad \text{1-2-1}$$

$$d\Sigma^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \qquad \text{1-2-2}$$

$$d\Omega^2 = d\theta^2 - \sin^2\theta d\phi^2 \qquad \text{1-2-3}$$

where k is the curvature constant and $a^2(t)$ is scale factor

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kC^2}{a^2} \qquad \text{1-2-4}$$

$$(\Omega^{-1} - 1)\rho a^2 = \frac{-3kc}{8\pi G} \qquad \text{1-2-5}$$

$$\rho_c = \frac{3H^2}{8\pi G} \qquad \text{1-2-6}$$

$$\Omega = \frac{\rho}{\rho_c} \quad 1\text{-}2\text{-}7$$

$$\Omega > 1,\ \rho > \rho_c \quad \Longrightarrow \text{ close universe} \quad 1\text{-}2\text{-}8$$

$$\Omega < 1, \rho < \rho_c \quad \Longrightarrow \text{ open universe} \quad 1\text{-}2\text{-}9$$

$$\Omega = 1, \rho = \rho_c \quad \Longrightarrow \text{ flat universe} \quad 1\text{-}2\text{-}10$$

However, NASA's Wilkinson Microwave Anisotropy Probe (WMAP) and ESA's Planck satellite result nailed down the cubature of space "flat" Euclidian.

The total density of the universe is

$$\Omega_{\text{totWMAP}} = 1.0027\ ^{+0.0039}_{-0.0038} \quad 1\text{-}2\text{-}11$$

$$\Omega_{\text{totPlanck}} = 1.0005^{+0.0065}_{-0.0060} \quad 1\text{-}2\text{-}12$$

Two satellites data play the key role in establishing the current standard model of cosmology. WMAP and Planck data are very well fit by a universe. Other cosmological data are also consistent, and together the data tightly constrain the standard model of the cosmology. The age of the universe is $(13.772 \pm 0.059)_{WMAP,}(13.798 \pm 0.037)_{Planck}$ billion years, as determined to better than 1% precision by the WMAP and Planck mission. The current expansions rate of the universe is $(69.32 \pm 0.80)_{WMAP,}(67.80 \pm 0.77)_{Planck}$ km s^{-1} Mpc^{-1}. The content of the universe presently consists of $(4.628\% \pm 0.093\%)_{WMAP,}(4.9\%\ )_{Planck}$ ordinary baryonic matter, $(24.02\%\ ^{+0.88\%}_{-0.87\%})_{WMAP,}(26.8\%\ )_{Planck}$ CDM that neither emits nor absorbs light, and $(71.35\%^{+0.95\%}_{-0.96\%})_{WMAP,}$ $(68.3\%\ )_{Planck}$ dark energy.

According to the prediction of general relativity, the present universe must have some curvature, because the density of the present universe by WMAP and Planck is

$$\rho_{\text{critical ord}} = \left(\frac{3H_0^2}{8\pi G}\right)(0.046)$$

$= (9.47\times 10^{-27}\ \text{kg}/m^3)(0.046)$

$= 4.356 \times 10^{-28}\ \text{kg/m}^3$ 1-2-13

Result show open universe

Based on prior successes of agreement of the full experimental effect of general relativity (e.g., the deflection of light by the sun, the precession of Mercury by the sun) produced with theoretical values calculated with ordinary matter like the sun's matter, the present universe's curved space is produced by the 0.046%~0.049% density of the ordinary matter of the present universe calculated according to Eq. 1-2-13 to satisfy theoretical consistency.Those results indicate that universe must be open. However, the curvature result of the WMAP and Planck data by Eq. 1-2-14 Eq. 1-2-15 is flat without cosmic inflation as

Ω_k=1-Ω_{totWMAP}=1-$1.0027\ ^{+0.0039}_{-0.0038}$ = - $0.0027\ ^{+0.0039}_{-0.0038}$ 1-2-14

Ω_k=1-$\Omega_{\text{totPlanck}}$=1-$1.0005^{+0.0065}_{-0.0060}$ = - $0.0005\ ^{+0.0065}_{-0.0066}$ 1-2-15

This result shows that the cubature of space time in the deflection of light by the sun, the precession of Mercury by the sun is not true. Cause of such phenomena is not curved space, because cosmic inflation cannot exist according to CFLE theory of quantum gravity (cf.§13.6). Therefore at the same time this result validates that the theory of relativity is no longer a consistent theory. Such contradictions to rationalize for Einstein's relativity to keep relativistic physicists who follow Einstein's relativity theory introduce cosmic inflation, dark matter, dark energy and cosmological constant.

However, from all of those facts most important point is that observed curvature of such universe is flat. Therefore field equation of general relativity doesn't need any more for curvature of space to calculate. Result of such theory is confusing as figure 1-2-1. According to present observation vacuum of universe don't have any kind of energy as dark energy for accelerating universe and cosmological constant. Because equivalence principle of general relativity don't allow such negative energy, under condition of general relativity cannot occur accelerating expansion of universe (cf.§20,§24). This means that concept of curved space, related field theory and ΛCDM model of standard cosmology is useless, meaningless and physically not true. Therefore theory of general relativity cannot predict future of universe as figure 1-2-1.

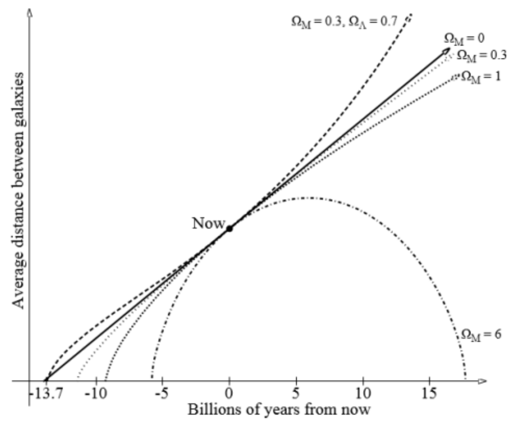

Figure 1-2-1

Furthermore concept of space–time continuum is not allowed in this universe (cf.§14,§24)

Therefore, one can argue that the results of the WMAP mission of NASA and Planck mission of ESA are the greatest victory of modern observations astronomy in the history of science. If the errors of relativistic physics can be improved, then it will be an honor of the WMAP team with NASA and Planck team with ESA.

Despite such clear results, cosmologists nowadays can only talk about this important defect, "the flatness problem of the universe." By examining the details of Friedman–Lemaitre-Robertson–Walker cosmology, it turns out that Ω, which is the ratio of the mass/energy density of the universe to its critical value, must have been very close to 1 during the earliest moments of cosmic history. If Ω were roughly near 1 today, for example, when the universe was 1 second old, it must have deviated from 1 by less than one part in ten thousand trillion ($1/10^{16}$); this is a fine-tuning problem. It is hard to understand how Ω could have been adjusted to such precision.

Another way to view the difficulty is as follows. If on the one hand Ω had been initially less than 1 but not precisely close to 1, the universe would have expanded and collapsed during its earliest stages of development. In other words, the age of the universe would have been a tiny fraction of a second. Since the universe is at least 10 billion years old, there must be a contradiction. If on the other hand Ω had been more than 1 but not precisely close to 1, the universe would have expanded extremely rapidly, cooling to a frigid temperature above absolute zero, but indiscernibly so. Since the universe is not that cold today, there must again be a contradiction. This problem is called the oldness problem of the cosmos. However, because of the results of the WMAP and Planck measurements, the flatness problem should be called the non-general relativity problem of the universe. The results of the WMAP and Planck measurements are crucial evidence that the theory of relativity has significant defects. Furthermore, in the 1960s, Roser Penrose and Steven Hawking mathematically proved that in the general theory of relativity there must always have existed a singularity, which is the point that has infinite density, the so-called singularity theorem. Following is a short presentation of the singularity theorem, listing some of its definitions, propositions, proofs, lemmas, and corollaries.[5] The starting condition is to have strong energy; that is,

$$T_{oo} + T_{oo} \geq 0 \qquad \text{1-2-16}$$

This is the most important energy condition (since gravity is attractive).

Some salient points of the theorem include, among others,

1. $R_{ab}K^aK^b \geq 0$, for every non-space-like vector K.

2. M contains no closed time-like curvature

$$R_{ab}K^aK^b < 0 \qquad \text{1-2-17}$$

3. Every non-space-like geodesic with a tangent vector contains a point at which

5. Readers should refer to standard physics texts for the full presentation of the singularity theorems, such as "*Singularity Theorems in Classical Relativity Theory*" by Vivek Narayanan, available at people.maths.ox.ac.uk/hausel/m392cr/narayanan.pdf.

$K_{[a}R_{b]cd[\varepsilon}K_{f]}K^cK^d \neq 0$ 1-2-18

4. There exists at least one of the following:

- a compact achronal set without edges
- a closed trapped surface
- the null geodesics from some point are eventually focussed, or the null geodesics from some closed two-surface are all conversing

Some basic definitions include that

- a *path* is a smooth map from a connected set in ℝ to M.
- a *curve* is the point set image of the path; if $t-\gamma_{(t)}$ is a path, the curve determined by $\gamma_{(t)}$ will be denoted γ.

The first proposition is that for all subsets, S ⊂ M:

1) $J^+(s) = I^+(s)$
2) $J^+(s) \subset \overline{I^+(s)}$ 1-2-19

…

The first lemma states that an achronal set boundary B is an achronal set.

Equations have also been established for the Strong Causality, the Alexandroff Topology, the Space of Causal Curve, and the Conjugate Point.

The Raychaudhuri equation for non-rotating flow implies

$\dot{\theta} \leq -\frac{1}{3}\theta^2$ 1-2-20

Written in the form $\theta^{-2}\dot{\theta} \leq -(\frac{1}{3})$,

$\int_{\lambda_1}^{\lambda_2} dx\theta^{-2}\dot{\theta} = \int_{\theta(\lambda_1)}^{\theta(\lambda_2)} d\theta\theta^{-2} = [-\theta^{-1}]_{\theta(\lambda_1)}^{\theta(\lambda_2)} \leq -\frac{1}{3}(\lambda_2 - \lambda_1)$ 1-2-21

Rearranging gives

$-\theta(\lambda_2) \geq [-\theta(\lambda_1)^{-1} - \frac{(\lambda_2-\lambda_1)}{3}]^{-1}$

$$\frac{d^2T}{d\alpha^2}|_{\alpha=0} = \int_a^b X^b T^c \nabla_c \left[X^a \nabla_a \left(\frac{T_{bb}}{f}\right)\right] dt + \int_a^b X^b T^a X^c R^d_{cab} \frac{T_d}{f} dt$$

$$= \int_a^b X^b \, (o)_b dt \qquad \textit{1-2-22}$$

…

Corollary 1: Let *F* be a future set, and γ a null geodesic on *dF*. Then γ contains no proper segment that has a pair of conjugate points

Theorem: The following are mutually inconsistent in any space–time:

a) There are no close trips.
b) Every endless causal geodesic contains a pair of conjugate points.
c) There is a future(past)-trapped set S in M.

In general, the singularity theorems show that under three assumptions, it follows that a singularity will occur. These assumptions are (1) a global condition on the space–time, allowing well-posed initial value problems; (2) a condition saying that in a region of space–time, there is a closed trapped surface; and (3) an energy condition of matter, saying that the energy flows are non-negative. These conditions guarantee the occurrence of singularities in the black hole.

Therefore the prediction of general relativity based on such singularities is its own downfall, and the related physical law consequently breaks down too. This is the clear result from the inevitability of singularity.

Another succinct description of this singularity theorem is given in the following Wikipedia excerpt.

> "The Penrose-Hawking singularity theorems are a set of results in general relativity that attempt to answer the question of when gravitation produces singularities. A singularity in the solution of the Einstein field equations is one of two things.
>
> (1) A situation where matter is forced to be compressed to a point.
>
> (2) A situation where certain light rays come from a region with infinite cubature.
>
> Space-like singularities are a feature of non-rotating uncharged black holes, whereas time-like singularities are those that occur in charged or rotating black-hole exact solutions. Both of these singularities have the property of geodesic incompleteness: that is, some light-paths or particle-paths cannot be extended beyond a certain proper-time or affine-parameter. It is still an open question whether time-like singularities ever occur in the interior of real charged or rotating black holes, or whether they are artefacts of high symmetry and disappear when realistic perturbations are added. The Penrose theorem guarantees that some sort of geodesic incompleteness occurs inside any black hole, whenever matter satisfies reasonable energy conditions. The energy condition required for the black-hole singularity theorem is weak: it states that light rays are always focused together by gravity, never drawn apart, and this holds whenever the energy of matter is non-negative.
>
> Hawking's singularity theorem is for the whole universe, and works backwards in time: it guarantees that the Big Bang has infinite density. The theorem is more restricted, since it only holds when matter obeys a stronger energy condition, called the dominant energy condition, which means that the energy is bigger than the pressure. All ordinary matter, with the exception of a vacuum expectations value of a scalar field, obeys this condition. During inflation, the universe violates the stronger dominant energy condition, and inflationary cosmologies avoid the initial Big-Bang singularity, rounding them out to a smooth beginning. In general relativity, a singularity is a place that objects or a light ray can reach in a finite time where the curvature become infinite or space-time stops being a manifold. Singularities can be found in all the black-hole space times, the Schwarzschild metric, the Reissner-Nordström metric, and the Kerr metric, and in all cosmological solutions that do not have a scalar field energy or a cosmological constant. One cannot predict what might have come "out" of a Big-Bang singularity in our past, or what will happen to an observer that falls "in" to a black-hole

singularity in the future, so a modification of the physical law is required.

Before Penrose’s observation, it was conceivable that singularities only form in contrived situations. For example, in the collapse of a star to form a black hole, if the star is spinning and thus processing some angular momentum, the centrifugal force may perhaps partly counteract gravity and keep a singularity from forming. The singularity theorems prove that this cannot happen, and that a singularity will always form once an event horizon forms. In the collapsing star example, since all matter and energy is a source of gravitational attraction in general relativity, the additional angular momentum only pulls the star together more strongly as it contracts: the part outside the event horizon eventually settles down to a Kerr black hole. The part inside the event horizon necessarily has a singularity somewhere.

The proof is somewhat constructive—it shows that the singularity can be found by following light-rays from a surface just inside the horizon. However, the proof does not reveal what type of singularity occurs (i.e, space-like, time-like, orbifold, jump discontinuity in the metric). It only guarantees that if one follows the time-like geodesics into the future, it is impossible for the boundary of the region they form to be generated by the null geodesics from the surface. This means that the boundary must either come from nowhere or the whole future ends at some finite extension.

An interesting “philosophical feature of general relativity is revealed by the singularity theorems. Because general relativity predicts the inevitable occurrence of singularities, the theory is not complete without a specification for what happens to matter that hits the singularity. In mathematics, there is a deep connection between the curvature of a manifold and its topology. The Bonnet–Myers theorem states that a complete Riemannian manifold, which has Ricci curvature everywhere, greater than a certain positive constant must be compact. The condition of positive Ricci curvature is most conveniently stated in the following way: for every geodetic there is a nearby initially parallel geodesic that will bend toward it when extended, and the two will intersect at some finite length. When two nearby parallel geodesics intersect, the extension of either one is no longer the shortest path between the endpoints. The reason is that two parallel geodesic paths necessarily collide after an extension of equal length, and if one path is followed to the intersection and then the other, you are connecting the endpoints by a non-geodesic path of equal length. This means that for a geodesic to be a shortest length path, it must never intersect neighboring parallel geodesics. Starting with a small sphere and sending out parallel geodesics from the boundary, assuming that the manifold has a Ricci curvature bounded below by a positive constant, none of the geodesics are shortest paths after a while, since they all collide with a neighbor. This means that after a certain amount of extension, all potentially new points have been reached. If all points in a connected manifold are at a finite geodesic distance from a small sphere, the manifold must be compact.

Penrose argued analogously in relativity. If null geodesics (the paths of light rays) are followed into the future, points in the future of the region are generated. If a point is on the boundary of the future of the region, it can only be reached by going at the speed of light, no slower, and so null geodesics include the entire boundary of the proper future of a region. When the null geodesics intersect, they are no longer on the boundary of the future; they are in the interior of the future. Therefore, all the null geodesics collide, and there is no boundary to the future.

In relativity, the Ricci curvature, which determines the collision properties of geodesics, is determined by the energy tensor, and its projection on the light ray is equal to the null-projection of the energy-momentum tensor and is always non-negative. This implies that the volume of a congruence of parallel null geodesics, once it starts decreasing, will reach zero in a finite time. Once the volume is zero, there is a collapse in some direction, so every geodesic intersects some neighbor. Penrose concluded that whenever there is a sphere where all the outgoing light rays are initially conversing, the boundary of the future of that region will end after a finite extension, because all the null geodesics will converge.

This is significant, because the outgoing light rays for any sphere inside of the horizon of a black hole solution are all converging, so the boundary of the future of this region is either compact or comes from nowhere. The future of the interior either ends after a finite extension, or has a boundary that is eventually generated by new light rays that cannot be traced back to the original sphere. The singularity theorems use the notion of the geodesic incompleteness as a stand-in for the presence of infinite curvatures. Geodesic incompleteness is the notion that there are geodesics, paths of observers through space-time, that can only be extended for a finite time as measured by an observer traveling along one, presumably at the pathology at which the laws of general relativity break down. Typically a singularity theorem has three conditions:

(1) An energy condition on the matter

(2) A condition on the global structure of space-time

(3) The condition that gravity is enough to strap a region

There are various possibilities for each condition, and each leads to different singularity theorems. A key tool in the formulation and proof of a singularity theorem is the Raychaudhuri equation, which describes the divergence θ of a congruence of geodesics, defined as the derivative of the log of the determinant of the congruence volume. The Raychaudhuri equation is

$$\dot{\theta} = -\sigma_{ab}\sigma^{ab} - \frac{1}{3}\theta^2 - E[X^{\rightarrow}]^a_{\ a} \qquad \text{1-2-23}$$

where σ_{ab} is the shear tensor of the congruence. The key point is that $E[X^{\rightarrow}]^a_{\ a}$ will be non-negative provided that the Einstein field equations and the weak energy condition hold and the geodesic congruence is null, or the strong energy condition holds and the geodesic congruence is time-like. When these conditions hold, the divergence becomes infinite at some finite value of the affine parameter. Thus all geodesics leaving a point will eventually reconverge after a finite time, provided the appropriate energy condition holds, a result also know as the focusing theorem. This is relevant for singularities, thanks to the following arguments:

(1) Suppose we have a space-time that is globally hyperbolic, and two points *p* and *q* that can be connected by a time-like or null curve. Then there exists a geodesic of maximal length connecting *p* and *q*. Call this geodesic γ.

(2) The geodesic γ can be varied to a longer curve if another geodesic from *p* intersects γ at another point, called a conjugate point.

(3) From the focusing theorem, we know that all geodesics from *p* have a conjugate point at finite values of the affine parameter. In particular, this is true for the geodesic of maximal length. However this is a contradiction.

One can therefore conclude that the space–time is geodesically incomplete. In general relativity, there are several versions of the Penrose-Hawking singularity theorem. Most versions state, roughly, that in a trapped null surface with non-negative energy density, there exist geodesics of finite length that cannot be extended. These theorems, strictly speaking, prove that there is at least one non-space-like geodesic that is only finitely extendible into the past, but there are cases in which the conditions of these theorems are obtained in such a way that all past-directed space-time paths terminate at a singularity.

Understanding the definition of the singularity is very important, since our universe has started from singularity, and all black holes contain a singularity. It is not reasonable to define a space-time singularity as a point where the metric tensor was undefined or was not suitably differentiable. However, the trouble with this is that one could simply cut out such points and say that the remaining manifold represents the whole of space-time, which would then be non-singular. Although we define the singularity of any black hole as a place where space-time is undefined in many occasions, such as the singularity of a Kerr black hole, note that in reality this is not true. Indeed, it would seem inappropriate to regard such singular points as being part of space-time, for the normal equations of physics would not hold them and it would be impossible to make any measurements.

The most important mathematical definition in the singularity theorem is the geodesic completeness. In the case of a manifold with a positive definite metric g, one can define a distance function $\rho(x, y)$, which is the greatest lower bound of the length of curves from x to y. The distance function is a metric in the topological sense; that is, a basis for the open sets of M is provided by the set $B(x, r)$ consisting of all points y that are elements to M such that $\rho(x, y) < r$. The pair (M, g) is said to be metrically complete if every Cauchy sequence with respect to the distance function ρ converges to a point in M. A Cauchy sequence is an infinite sequence of point $x(n)$ such that for any $\varepsilon > 0$ there is a number N where $\rho(x(n), x(m)) < \varepsilon$ whenever n and m are greater than N. Therefore, m-completeness implies geodesic completeness; that is, every geodesic can be extended to an arbitrary value of its affine parameter. This is because if (M, g) is m-completeness, every differentiable of finite length has an endpoint.

Time-like geodesic incompleteness has an important property; there could be freely moving observers or particles whose histories did not exist after a finite interval of proper time. Thus, it is appropriate to regard such a space as being singular. Even though the affine parameter on a null geodesic does not have quite the same physical significance as proper time does on time-like geodesics, one should regard a null geodesically incomplete space-time as being singular, both because null geodesics are the histories of zero rest mass particles and because there are some examples that one would think of as being singular but which are time-like and not null geodesically complete. Therefore, if a space-time is time-like or null geodesically complete, we shall say that it has a singularity.

Theorems that were provided by Penrose in 1965 are very important. Before he provided the theorem to be proven, it was possible to hope that collapse to Schwarzschild singularity was an artefact of spherical symmetry, and typical geometries would remain non-singular. However, they provided a fact that once a collapse reaches a certain point, evolution to a singularity is inevitable.” [6]

6. Source: http://en.wikipedia.org/wiki/Penrose-Hawking singularity theorems (accessed June 2011).

Such theoretical defect is inconsistent with quantum theory and more serious than thought, because it is impossible to have the world partly classical (a world with mathematical infinity) and partly quantum mechanical (a world mathematically renormalizable). Consequently, the theory of relativity must be modified, because it is inevitable that singularity ruins the law of fundamental physics and causes unprecedented crises even in basic physics.

1.3 Cause of the Inconsistency

If one analyzes the simplest case of Figures 1-3-1 and 1-3-2, which can be applied to the equivalence principle, the cause of the inconsistency of the relativity theory can be very clearly seen and simply found.

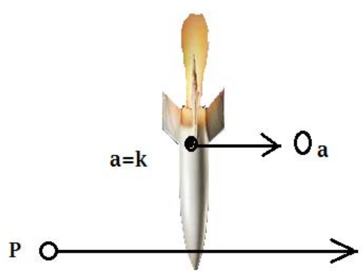

Figure 1-3-1

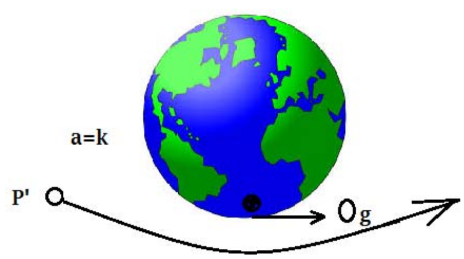

Figure 1-3-2

Figure 1-3-1 shows that object P moves with uniform velocity $v = K$ from left to right. When the trajectory of object P is observed by observer O_a who is at rest in a rocket that flies with uniform acceleration ($a = k$), the trajectory of object P must be the same trajectory of object P′ in Figure 1-3-2, which moves through the gravitational field at uniform acceleration ($a = k$), according to the equivalence principle. Therefore, the resting observer in gravitational field O_g in Figure 1-3-2 obtains the same result as observerO_a.

In this case especially, the improvement point is that the special relativity theory is established not only in inertial frame, but also strictly in the system of acceleration. The physical and mathematical justifications of such general establishment of the special theory of relativity are founded and proven during the integration process for energy as follows:

$$KE = \int Fds \qquad 1\text{-}3\text{-}1$$

$$F = \frac{d}{dt}\left(\frac{m_0 V}{\sqrt{1-\frac{v^2}{c^2}}}\right) = m\frac{dv}{dt} + V\frac{dm}{dt} = ma + vi \qquad 1\text{-}3\text{-}2$$

$$KE = \int_0^s Fds = \int_0^s \frac{d}{dt}(mv)ds = \int_0^{mv} \frac{ds}{dt} d(mv)$$

$$= \int_0^{mv} vd(mv) = \int_0^v vd\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= \frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 \int_0^v \frac{vdv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}} + m_0 c^2 \left|\sqrt{1-\frac{v^2}{c^2}}\right|_0^v$$

$$= \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2 = (m - m_0)^2 c^2 \qquad 1\text{-}3\text{-}3$$

$$\therefore E = mc^2 \qquad 1\text{-}3\text{-}4$$

$$F = \frac{d}{dt}\left(\frac{m_0 V}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad 1\text{-}3\text{-}5$$

Process 1-3-5 shows that the special relativistic moment is differentiated, and for “special relativistic moment to differentiate” it means none other than that the special relativity theory applies to the system of acceleration. The physical and mathematical justifications of this integration and its results have been proven again and again (e.g., the atom bomb), and these facts guarantee that the special relativity theory can be established in the system of acceleration too. Accordingly, Figure 1.3.2 demonstrates the special theory as applied to the system of acceleration.

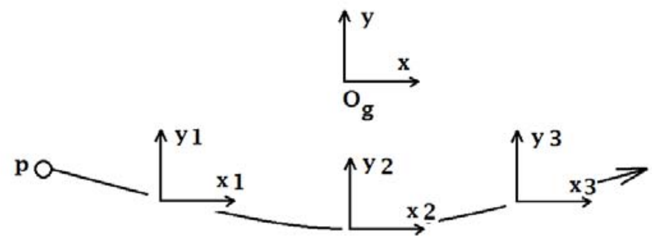

Figure 1-3-3

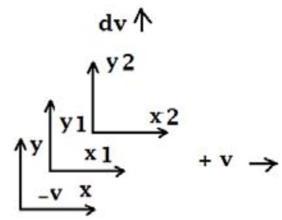

Figure 1-3-4

Figure 1-3-3 shows the situation as seen by an observer in the gravitational rest frame XY. Object P is momentarily at rest in the frame X_1Y_1 at the instant t_1, and momentarily at rest in the frame X_2Y_2 at the slightly later instant t_2. Both the axes of XY and of X_2Y_2 have been constructed parallel to the axes ofX_1Y_1, as seen by an observer in X_1Y_1. Figure 1-3-4 shows XY, X_1Y_1, and X_2Y_2 from the point of view of the observer in X_1Y_1. Since the object P is moving with velocity *v* relative to the source of the gravitational field, the axes XY are moving with velocity $-v$ in the direction of the negative X_1 axis relative to X_1Y_1. As seen in X_1Y_1, the object P is accelerating toward the source of the gravitational field with acceleration "a" in the direction of the positive Y_1 axis. If the time interval $(t_2 - t_1)$ is very small, the change in velocity *v* of the object P in that interval will be

$$dv = a(t_2 - t_1) = adt \qquad \text{1-3-6}$$

This will be the velocity of X_2Y_2 as seen by X_1Y_1. Now use the relativistic velocity transformation equations to evaluate the components of U_a, the velocity of X_2Y_2 as seen by XY. These give

$$U_{ax} = \frac{dv_x - v_x}{1 - \frac{v_x dv_x}{c^2}} = \frac{0+v}{1} = v \qquad \text{1-3-7}$$

$$U_{ay} = \frac{dv_y\sqrt{1-\frac{v_x^2}{c^2}}}{1-\frac{v_x dv_x}{c^2}} = \frac{dv\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{o.dv_x}{c^2}} = dv\sqrt{1-\frac{v^2}{c^2}} \qquad \text{1-3-8}$$

Using the same transformation equations to evaluate the components of U_b, the velocity of XY as seen by X_2Y_2 can obtain

$$U_{\rm bx}=\frac{v_x\sqrt{1-\frac{dv_y^2}{c^2}}}{1-\frac{dv_y v_y}{c^2}}=\frac{-v\sqrt{1-\frac{dv^2}{c^2}}}{1-\frac{dv.\text{o}}{c^2}}=-v\sqrt{1-\frac{dv^2}{c^2}} \qquad \text{1-3-9}$$

$$U_{\rm by}=\frac{v_y-dv_y}{1-\frac{dv_y v_y}{c^2}}=\frac{\text{o}-dv_y}{1-\frac{dv_y.\text{o}}{c^2}}=-dv \qquad \text{1-3-10}$$

Now, using these results, the case of deflection of the light by the sun can be obtained.

$$U_{\rm ax}=c, \quad U_{ay}=dv\sqrt{1-\frac{c^2}{c^2}}=dv.o=0 \qquad \text{1-3-11}$$

$$U_{\rm bx}=-C\sqrt{1-\frac{dv^2}{c^2}}=-C, \quad U_{\rm by}=-dv \qquad \text{1-3-12}$$

Here, the important point is that the Y component of velocity appeared for one frame but not for the other frame. These results show that there is no relativistic movement between the two frames. In other words the principle of relativity is likely broken here. Thus one can easily and quickly recognize that the mortal inconsistency of the relativity theory starts with this scene.